BE/APh 161: Physical Biology of the Cell, Winter 2014
SciPy `odeint` tutorial

To illustrate how to use Python and its very useful modules NumPy and SciPy to solve systems of ODEs, we will use the Lotka-Volterra model as an example. The Lotka-Volterra model decsribes the dynamics of a predator-prey pair. If we let N be the number of prey individuals and P by the number of predators, the Lotka-Volterra model reads

$$\frac{\mathrm{d}N}{\mathrm{d}t} = aN - bNP \tag{0.1}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = cNP - dP. \tag{0.2}$$

To get a feel for what the terms mean, we'll describe each. First, aN is the growth rate of prey. It essentially assumes the prey have a steady food source, and they grow in population with rate constant a. The term $-bNP$ is the rate at which prey are killed off by predators. This is a pairwise interaction: the predator and prey must meet for the prey to perish. It perishes with rate constant b. Similarly, the predators need to be eating prey to grow in population, and this is captured in the cNP term. Finally, the $-dP$ term describes the dying off of predators in the absense of prey.

We can nondimensionalize this problem and thereby reduce the number of parameters from four to one. We define dimensionless time by $\tau = at$, dimensionless prey population by $u = cN/d$, dimensionless predator population by $v = bP/a$. Then, the Lotka-Volterra model reads

$$\frac{\mathrm{d}u}{\mathrm{d}\tau} = u(1 - v) \tag{0.3}$$

$$\frac{\mathrm{d}v}{\mathrm{d}\tau} = \alpha v(u - 1), \tag{0.4}$$

where $\alpha = d/a$. These are the coupled ODEs we seek to solve. They are nonlinear, so we will resort to numerical integration. We will use the `odeint` function in the `scipy.integrate` module. Briefly, the `odeint` function is a wrapper around the `LSODA` integrator of `ODEPACK`, which switches between implicit and explicit time stepping schemes, depending on the local stiffness of the system. This means that it is quite robust; you can through lots of systems of ODEs at it, and it will perform well.

There are many ways to install NumPy and SciPy, but installing Canopy from Enthought is quite easy. You can get more information about this Python distribution here:
http://www.enthought.com/products/canopy/

Working through the code in `lotka_volterra.py` will help you understand how to do the numerical integration. This is the most basic way to use the `odeint` module, but will suffice for many of the purposes of our course.

To run the code, you can use iPython (recommended), or invoke python from the command line as `python lotka_volterra.py`. To invoke an iPython notebook, enter "`ipython notebook &`" on the command line, provided you have installed Enthought Canopy. You can then follow the onscreen instructions to run the `lotka_volterra.py` scriptin an iPython notebook in a web browswer.