BE/APh 161: Physical Biology of the Cell, Winter 2014
Homework #6
Due at the start of lecture, 1PM, February 26, 2014.

Problem 6.1 (Comments on *Cell Biology by the Numbers* part 6, 10 pts).
We continue our reading of *CBBTN*. This time, please read chapter 6, pages 297–329, and send comments about two vignettes. Remember to email your answers to me and the TAs and indicate whether you would like to be anonymous when I send the comments to the book's authors. Also, please either send your responses as text in an email or as a PDF. Do not send MS Word documents.

Problem 6.2 (Swimming at low Re (based on *PBoC2* problem 12.6(a)), 10 pts).
E. coli swims at about 20 µm/s by rotating a bundle of helical flagella. If the motors were to turn 10 times faster than normal, what would the swimming speed of *E. coli* be? If the fluid environment were made 10 times more viscous, but the motors were to turn at the same rate, what would the swimming speed be? How does the power output of the motor change in these two hypothetical situations?

Problem 6.3 (Microtubule buckling in a cell, 15 pts).
When we studied filament buckling, we found the critical force for buckling by writing the total energy of a rod being compressed from the ends. The expression for the energy was a function of the radius of curvature R of the bent configuration. We found the applied force was enough to buckle the filament when the minimal energy occurred at finite R. In doing that calculation, we assumed that the rod buckled in a simple arc. This is the lowest order buckling mode. A higher order mode would have a sinusoidal looking shape.

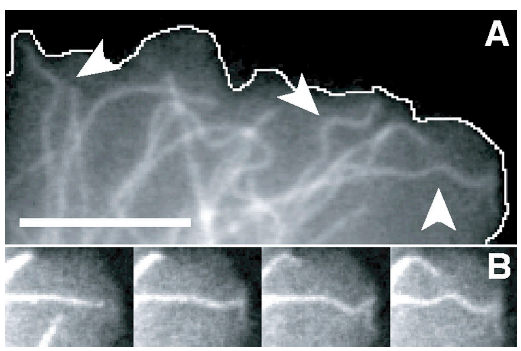

Figure 1: A) Depiction of EGFP-labeled microtubules pressing against a cell boundary in a Cos7 cell. B) Time sequence (5 seconds between images) of a microtubule polymerizing into the edge of the cell and buckling. Scale bar, 5 µm. Figure from Brangwynne, et al., *J. Cell. Biol.*, **173**, 733–741, 2006.

a) Make an argument based on dimensional analysis why we would expect the lowest order mode for buckling of a filament in solution.

b) Now imagine that the filament we are considering is embedded in an elastic meshwork, like we might see in a cell. Experimentally (see Fig. 1), we observe small wavelength buckling of microtubules as they press against the cell boundary. Use dimensional analysis and physical reasoning based on Figure 1 to estimate the critical buckling force and wavelength λ of the buckling if a rod is embedded in an elastic meshwork that has Young's modulus E.

c) Later in the course, we will find that $E \approx 1$ kPa. Given that this is the case, predict the bending wavelength of a microtubule and compare it to what is seen in Figure 1. Also compare the critical bending forces for a microtubule 10 µm in length embedded in water vs. embedded in the elastic meshwork of a cell.

Problem 6.4 (Flexural rigidity of biopolymers, adapted from problem 10.2 of *PBoC2*, 20 points)**.**

a) Recall that the flexural rigidity of a filament is $K_{\text{eff}} = EI$, where E is the Young's modulus and I is the geometric moment of inertia defined in lecture. We also saw that the persistence length is given by $\xi_p = EI/k_BT$. Given the persistence lengths of DNA, actin filaments, and microtubules (check your lecture notes or BioNumbers), estimate their respective Young's moduli by computing the moment of inertia. You can look up geometric information about the filaments in *PBoC2* sections 2.2.3 and 10.5.1.

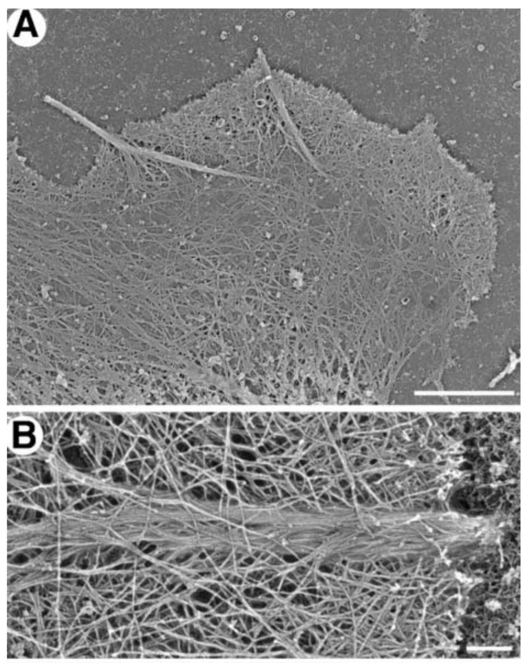

Figure 2: A) Electron micrograph of a B16F1 cell with a few peripherally located filopodia. Scale bar, 5 µm. B) A close-up of one of the filopodia. Scale bar, 1 µm. Image taken from Mejillano, et al., *Cell*, **118**, 363–373, 2004.

b) Filopodia are protrusions of bundled actin filaments often found in adherent cells. They push against the cell membrane. The membrane pushes back on the filopodium with a force of

$$F = 2\pi r\gamma, \tag{6.1}$$

where r is the radius of the end of the filopodium and γ is the surface tension of the membrane. We will later discuss how to measure the surface tension of the membrane, but for now, we will take it to be $\gamma \approx 0.035$ pN/nm. We will assume that the filopodium consists of approximately 30 filaments. We will now investigate how long the filopodium can protrude before it buckles, considering two limits.

i) First, we assume that the filaments in the filopodium are not crosslinked. Find the length L that the filopodium can protrude before buckling.

ii) Now, consider the limit where the filaments in the filopodium are very tightly crosslinked, so tightly crosslinked that the filopodium can be considered a solid rod. Find the length L_{cl} that the crosslinked filopodium can protrude before buckling.

iii) In general, what is L_{cl}/L as a function of N, the number of filaments in the filopodium?

Problem 6.5 (Polymerization as a force generator, 15 pts)**.**
Imagine an actin filament is polymerizing against a compressive force. This might be the case if it polymerizes against a membrane, which can deform but nonetheless provides a compressive force on the filament.

a) Let K_{d} be the dissociation constant for binding an additional actin monomer to the end of an actin filament, as defined in lecture. Let δ be the increased length of an actin filament as a result of adding one monomer. Show that at equilibrium, the filament can exert a force of

$$F_{\text{eq}} = \frac{k_B T}{\delta} \ln \frac{c_1}{K_{\text{d}}}, \tag{6.2}$$

where c_1 is the concentration of actin monomer. Estimate F_{eq} for actin, given that cells typically have $c_1 \approx 20$ µM. *Hint*: It might help to think about states and weights.

b) What is the maximal length of a filament such that it can polymerize against a compressive load without buckling? Derive an analytical expression and then plug in numbers for actin.

c) F_{eq} is the maximal force a filament can exert against a compressive load, as at equilibrium the polymerization force balances the compressive load. Experimentally, it is often the case that this force is never achieved, with polymerization essentially stalling at forces smaller than F_{eq}. Provide an intuitive explanation as to why this might be the case.