BE/APh 161: Physical Biology of the Cell, Winter 2016
Homework #2
Due at the start of lecture, 1PM, January 20, 2016.

Problem 2.1 (Mathematizing a cartoon for ciliar growth, 50 pts)**.**
Another model for flagellar/ciliar growth was proposed in Howard, et al., *Nat. Rev. Mol. Biol.*, **12**, 393–398, 2011. The cartoon is shown in Fig. 1. The text from the caption in the paper reads as follows. "Schematic of an advection-reaction model, a hypothetical mechanism for the length control of cilia and microvilli. Cargoes, for example growth factors, carried along cilia and microvilli are inactivated over time by phosphatases, which may provide a length-dependent signal to the growing tip."

Let $c(x,t)$ be the concentration of active growth factors in the cilium and let $\ell(t)$ be the length of the cilium.

a) Write down a set of differential equations to describe the dynamics of c and ℓ. If you like, you may assume a constant number of cargo-carrying motors as we did in lecture for the *Chlamydomonas* flagella, or you may assume that the density of motors is constant. Be sure to state any other assumptions or decisions you made in mathematizing the cartoon.

b) Nondimensionalize your dynamical equation(s) and comment on any physical insight this procedure provides.

c) If you can, solve for $\ell(t)$ analytically. If not, solve it numerically. Use your solution to also plot the growth rate, $\mathrm{d}\ell/\mathrm{d}t$, over time.

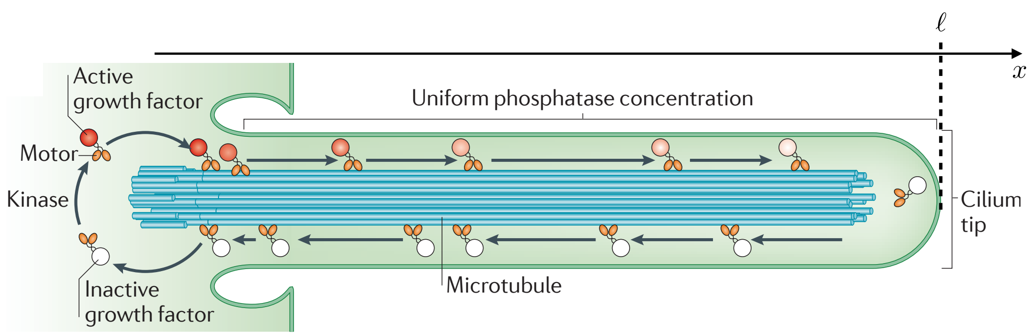

Figure 1: A cartoon describing a possible mechanism for ciliar growth adapted from Howard, et al., *Nature Rev. Mol. Cell Biol.*, **12**, 393–398, 2011. The text from the caption in the paper reads as follows. "Schematic of an advection-reaction model, a hypothetical mechanism for the length control of cilia and microvilli. Cargoes, for example growth factors, carried along cilia and microvilli are inactivated over time by phosphatases, which may provide a length-dependent signal to the growing tip."

Problem 2.2 (Growth curves, based loosely on page 103 of *PBoC2*, 45 pts)**.**
In chapter 3 of *PBoC2*, the authors considered growth curves of *E. coli*. The logistic equation is written as

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1 - \frac{N}{K}\right), \tag{2.1}$$

where N is the number of bacteria, r is the growth rate, and K is the carrying capacity, or the maximum number of bacteria that can be present and still have growth. In the analysis in *PBoC2*, the values of r and K were assumed to be constant. For bacteria growing in media, r and K could also be functions of the the concentration of food in the media, which we will call $F(N, t)$ (not to be confused with $f(N, t)$ in equation 3.10 in *PBoC2*).

Since we will be taking N as a continuous variable, and since F is a concentration, we can write the logistic equation in terms of concentration of bacteria, c, by dividing the entire equation by the volume V of the vessel containing the medium.

$$\frac{\mathrm{d}c}{\mathrm{d}t} = rc\left(1 - \frac{c}{K'}\right), \tag{2.2}$$

where $K' \equiv K/V$. We will drop the prime henceforth for notational convenience.

a) Write down an expression for $\mathrm{d}F/\mathrm{d}t$. You should try to keep your expression simple. Give your reasoning for how you chose this expression.

b) Sketch functional forms that you think are reasonable for $r(F)$ and $K(F)$. Again, try to keep them simple.

c) Based on what you know about bacterial growth, give reasonable values of the parameters you defined in your expressions for $\mathrm{d}F/\mathrm{d}t$, $r(F)$ and $K(F)$. Also give reasonable values for the initial bacteria concentration, c_0, and the initial food concentration, F_0. Explain how you came up with these values. *Hint*: Working through problem 2.5 of *PBoC2* will help you.

d) Solve the differential equations (numerically or analytically) and plot the results. You can use whatever numerical integration software you like. If you would like to use Python with NumPy/SciPy, the Jupyter notebook accompanying lecture 3 might serve as a useful reference.

e) Explain the shape of the curves.

f) Comment on any enhancements you would propose to this model for bacterial growth.

Problem 2.3 (Boltzmann's grave, 5 pts)**.**
Boltzmann's grave is in Zentralfriedhoff in Vienna, a beautiful cemetery that also contains the graves of some of the world's greatest composers, including Beethoven, Brahms, Schubert, Strauss, Ligeti, and Falco. On Boltzmann's impressive grave stone is inscribed the equation

$$S = k \log W. \tag{2.3}$$

Here, S is entropy, k is the Boltzmann constant, log refers to the natural logarithm, and W is the number of microstates. In class on January 13, we derived the famous Boltzmann distribution by maximizing the Shannon entropy, given that we knew an average energy of our system of interest. Derive the equation on Boltzmann's grave using the same technique. To do so, assume we do not know anything about the energy of the system.