BE/APh 161: Physical Biology of the Cell, Winter 2018
Homework #9
Due at the start of lecture, 2:30 PM, March 7, 2018.

Problem 9.1 (Optical cell stretching, 60 pts).
We briefly discussed optical cell stretchers in lecture. Optical cell stretchers work by taking advantage of the difference in index of refraction between a cell and the surrounding solution to trap a free cell in two counter-propagating laser beams. The power of the laser is then increased to exert stress and elongate the trapped cell. The induced stress is proportional to the laser power. The constant of proportionality, F_G is dependent on geometry and cannot be ascertained. The deformation (strain) is measured by taking images with a light microscope. The process is illustrated in Figure 1. In this way, the mechanical properties of an entire cell can be measured.

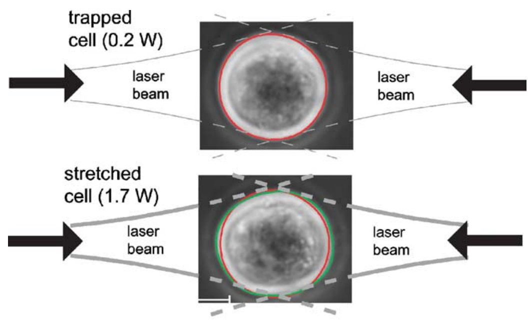

Figure 1: Schematic of an optical stretcher. The cell stretches along the axis parallel to the laser beams. The strain is given by the fractional change of the diameter of the cell along the stretching axis. Figure take from Wottawah, et al., *Acta Biomaterialia*, **1**, 263–271, 2005.

This technique was used to assess the mechanical properties of two mammalian cell types, 3T3 and SVT2 (which have reduced actin), in Wottawah, et al., *PRL*, **94**, 098103, 2005. In this work, the authors performed a stress step experiment in which a constant stress σ_0 was applied at $t = 0$, as in lecture. The stress was set back to zero at time $t = t_1$. The authors can obtain the creep compliance from this measurement.

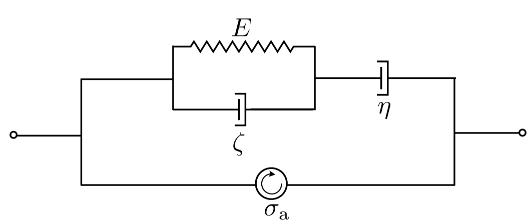

Figure 2: Schematic of an active Jeffreys fluid.

a) Derive an expression for the strain in the stress step experiment if we model the cell as an active Jeffreys fluid as in Figure 2. The stress step can be described mathematically as

$$\sigma(t) = F_G\,\sigma_0\,\theta(t)\,\theta(t_1 - t), \tag{9.1}$$

where $\theta(t)$ is the Heaviside step function. Assume the active stress is constant, given by σ_{a}.

b) The authors perform curve fits of the expression you derived in part (a) to get values for the parameters of the cell. Explain why they cannot independently measure E, η, and ζ, but only products thereof. Can a constant active stress be detected in this experiment?

c) The authors then use the curve fit parameters to compute the storage and loss moduli (E' and E'') of the cell. Derive expressions for the storage and loss moduli from the fit parameters. (*Note*: These reported storage and loss moduli are dependent on choosing a model for the viscoelastic behavior of the cell. This is not ideal, but is apparently a necessity due to experimental constraints.)